Monopole Vacuum in Non-Abelian Theories

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Abstract

It is shown that, in the theory of interacting Yang -Mills fields and a Higgs field, there is a topological degeneracy of Bogomol'nyi-Prasad-Sommerfield (BPS) monopoles and that there arises, in this case, a chromoelectric monopole characterized by a new topological variable that describes transitions between topological states of the monopole in the Minkowski space (in just the same way as an instanton describes such transitions in the Euclidean space). The limit of an infinitely large mass of the Higgs field at a finite density of the BPS monopole is considered as a model of the stable vacuum in the pure Yang-Mills theory. It is shown that, in QCD, such a monopole vacuum may lead to a rising potential, a topological confinement and an additional mass of the η_0 meson. The relationship between the result obtained here for the generating functional of perturbation theory and Faddeev-Popov integral is discussed.

1 Introduction and formulation of problem.

The problem of choosing, in a non-Abelian theory, a physical vacuum and variables that adequately reflect the topological properties of the manifold of initial data for non-Abelian fields [1, 2, 3] in the Minkowski space is still considered as one of the most important problems in these realms. There are reasons to believe that solving this problem will contribute to obtaining, within QCD, a deeper insight into the nature of the confinement, hadronization and spontaneous break-down of scale invariance in the infrared region.

The present study is devoted to employing monopole solutions [4] to the equations of a non-Abelian theory for the purpose to construct a model of a topologically invariant vacuum of the Yang-Mills (YM) theory in the Minkowski space. The respective Lagrangian density of that theory has the form

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a, \tag{1.1}$$

where

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A_{b\mu} A_{c\nu}. \tag{1.2}$$

¿From the mathematical point of view, the main problem in a theory of gauge fields is to find general solutions to the equations of that theory,

$$D^{ab}_\mu G^{\mu\nu}_b = 0 \quad (D^{ab}_\mu = \delta^{ab}\partial_\mu + g\epsilon^{acb} A_{c\mu}), \tag{1.3}$$

and to construct, in order to describe processes in terms of probability amplitudes normalized per time and spatial-volume units, the generating functional for the appropriate S-matrix in the class of functions such that the energy density is finite [5, 6, 7]. In QED such functions have the $O(1/r^{1+m})$ behaviour at the spatial infinity. They are referred to as monopoles if $m = 0$ and as multipoles if $m > 0$.

Solving differential equations in theoretical physics presumes specifying initial data. Such initial data are measured by a set of physical instruments with which one associates a reference frame. In the present study we will consider reference frames that are determined by the timelike unit vector $l^{(0)}_\mu = (1, 0, 0, 0)$ and various Lorentz transformations of it, $l^{(1)}_\mu$.

There are two types of the groups of transformations of the differential equations of a gauge theory. There are relativistic transformations, which change initial data (i.e. the appropriate reference frame) and gauge transformations,

$$\hat{A}^u_\mu(t;\mathbf{x}) := u(t;\mathbf{x})(\hat{A}_\mu + \partial_\mu)u^{-1}(t,\mathbf{x}); \quad \hat{A}_\mu = g\frac{\tau^a}{2i} A_{a\mu}; \tag{1.4}$$

which are associated with the gauge of physical fields and which do not affect the readings of an instrument.

The set of equations (1.3) is referred to as a relativistic covariant set of equations if the total manifold of its solutions for each specific reference frame coincides with its counterpart for any other reference frame (or isomorphic it) [7, 8, 9, 10, 11, 12].

In each reference frame the set of all the equations is split into the equations of motion: $D^{ab}_\mu G^{\mu i}_b = 0$ $(i = 1, 2, 3)$; to solve these equations, it is necessary to measure

initial data, and the constraint equations $D^{ab}_{\mu}G^{\mu 0}_{b}=0$, which relate initial data for the spatial components of the fields involved to initial data for their temporal components. [1]

In view of the said, Dirac [14] and, after him, other authors of the first classic studies devoted to quantizing gauge theories (see [15, 16]) removed temporal field components by gauge transformations. In our case, such a transformation,

$$\hat{A}^D_k = v(\mathbf{x})T\exp\{\int\limits_{t_0}^{t} d\bar{t}\hat{A}_0(\bar{t},\mathbf{x})\}(\hat{A}_k+\partial_k)[v(\mathbf{x})T\exp\{\int\limits_{t_0}^{t} d\bar{t}\hat{A}_0(\bar{t},\mathbf{x})\}]^{-1} \tag{1.5}$$

(here the symbol T denotes the time ordering of the matrices under the exponential sign), specifies a non-Abelian analogue of Dirac's variables: apart from arbitrary stationary matrices $v(\mathbf{x})$, which are considered as initial data, at the time instant t_0, for solving the equation

$$U(A_0+\partial)U^{-1}=0.$$

At the level of Dirac's variables, Lorentz transformations of original fields become nonlinear, while the group of gauge transformations reduces to a group of stationary transformations that specify the degeneracy of initial data for physical fields (including the classical vacuum $A_0=A_i=0$, which is defined as the zero-energy state). By the gauge fixation one means, in this case, the presetting initial data in a perturbation theory as the transversality condition [9]-[12].

In the YM theory the set of stationary gauge transformations is the set of three-dimensional paths in the space of the $SU_c(2)$ group that are broken down into topological manifolds characterized by integers (*Pontryagin degrees of a map*):

$$\begin{aligned}\mathcal{N}[n] &= -\frac{1}{24\pi^2}\int d^3x\epsilon^{ijk}Tr[v^{(n)}(\mathbf{x})\partial_i v^{(n)}(\mathbf{x})^{-1}v^{(n)}(\mathbf{x})\partial_j v^{(n)}(\mathbf{x})^{-1}v^{(n)}(\mathbf{x})\partial_k v^{(n)}(\mathbf{x})^{-1}]\\ &= n\in\mathbf{Z}.\end{aligned} \tag{1.6}$$

[1]The both sets may intersect, as far as the constraint equations are imposed onto the generalized velocities, i.e. onto the first time derivatives of gauge fields (see, e.g., the theory (1.11)- (1.12) in [13]). An important example of such intersection is the equation on the temporal component of a YM field. It is the motion equation and constraint simultaneously. We shall utilize this fact in the present statement.

In the "pure" YM theory (see [13], §16) temporal components of YM fields occupy a particular position, since they have no nonzero canonical momenta:

$$E_0=\partial\mathcal{L}/\partial(\partial_0 A_0)=0,$$

that contradict the commutation relations and Heisenberg's uncertainty principle [12].

Such situation takes place in the gauge theories involving [13] degenerated Hessian matrices

$$M_{ab}=\frac{\partial^2\mathcal{L}}{\partial\dot{q}^a\partial\dot{q}^b},$$

with q^i being appropriate degrees of freedom in the gauge theory considered and $\dot{q}^i$ being their time derivatives. Maxwell electrodynamics and non-Abelian models are patterns of that theories.

Any degree of a map indicates how many times the three-dimensional path $v(\mathbf{x})$ goes about the $SU_c(2)$ group manifold as the coordinate $\mathbf{x_i}$ runs over the entire three-dimensional space where this coordinate is specified.

The condition (1.6) means that the complete set of three-dimensional paths has the homotopic group $\pi_3(SU_c(2)) = \mathbf{Z}$ and that all the fields $v^{(n)}\partial_i v^{(n)-1}$ are given in the class of functions for which the integral (1.6) takes finite (or countable) values. This is the class of monopole functions [1, 2]. Naturally, the fields $A_i^D(t, \mathbf{x})$ themselves also must belong to this class of monopole functions and have the $O(1/r^{1+m})$, $m > 0$, asymptotic behaviour.

Thus our objective is to quantize non-Abelian fields in the class of monopole functions that involve a topological degeneracy. Such a quantization presumes the choice of Dirac's variables in which this degeneracy occurs.

The primary Hamiltonian quantization of non-Abelian gauge theories in terms of Dirac's variables without allowing for their topological degeneracy was due to Schwinger [10], who proved the relativistic covariance of the transverse gauge at the level of commutation relations for the generators of the algebra of the Poincare group that were constructed in the above class of functions. This Hamiltonian quantization of non-Abelian fields was then reproduced by Faddeev [17], who employed the method of a path integral $Z_{l^{(0)}}$ explicitly dependent on a reference frame. There was shown in [18] that the relativistic transformation at the level of fundamental operator quantization by Schwinger [10] corresponds to the relativistic transformation $l^{(0)} \to l^{(1)}$ of the time axis on which depends the path integral $Z_{l^{(0)}}$. The dependence of this integral on a reference frame is called an implicit relativistic covariance. [2]

That the path integral $Z_{l^{(0)}}$ is independent on a reference frame for on-shell amplitudes of elementary-particle scattering was first discovered by Feynman [22] and was proven by Faddeev [17] as a validation of the heuristic Faddeev-Popov (FP) path integral [23]. This integral was proposed as a generating functional of unitary perturbation theory for any gauges, including those that are independent of a reference frame. Schwinger noticed that gauges that are independent of a reference frame may be physically inadequate to the fundamental operator quantization; i.e. they may distort the spectrum of the original system. [3]

In the present study we verify this Schwinger's statement in a non-Abelian theory, answering the question concerning the spectrum of a theory quantized in the class of monopole functions that involves a topological degeneracy of initial data and the question concerning the relationship between the fundamental quantization and the heuristic FP integral in a gauge that is independent on a reference frame.

The ensuing exposition is organized as follows. Section 2 is devoted to describing the topological degeneracy of known monopole solutions and to considering zero modes of the constraint equation. In Section 3 we examine the limiting transition to the "pure" YM theory having a monopole vacuum. In Section 4 we analyse the $U(1)$-problem. In the Conclusion we discuss the connection with the FP integral.

[2] The choice of the time axis for such an integral was discussed in the works [19, 20, 21].

[3] "We reject all Lorentz gauge formulations as unsuited to the role of providing the fundamental operator quantization"[10].

2 Topological degeneracy of BPS monopole.

Let us consider the well-known example of interacting YM and scalar Higgs fields for the case where there is a spontaneous breakdown of the initial $SU(2)$ gauge symmetry. This situation is described by the Lagrangian density [4]

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a + \frac{1}{2}(D_\mu\phi^a)(D^\mu\phi_a) - \frac{\lambda}{4}(\frac{m^2}{\lambda} - \phi^2)^2, \tag{2.1}$$

where

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A_{b\mu}A_{c\nu} \tag{2.2}$$

is the strength tensor of our non-Abelian field and ϕ^a ($a = 1, 2, 3$) is a scalar field forming a triplet of the adjoin representation of the $SU(2)$ gauge group. The potential energy in the YM model considered depends on the square of the isotopic vector ϕ^a, while the covariant derivative of a YM field has the form

$$D^\mu\phi_a = \partial^\mu\phi_a + g\epsilon_{abc}A^{b\mu}\phi^c. \tag{2.3}$$

The Lagrangian density $\mathcal{L}$ possesses a manifest gauge invariance under transformations of the $SU(2)$ group.

The classical vacuum is defined as the asymptotic solution

$$r = |\mathbf{x}| \to \infty, \quad E \to \min E, \tag{2.4}$$

providing the minimum of the field energy E.

For $m^2 \geq 0$ and $\lambda \geq 0$, the Minkowskian YM vacuum loses its initial $SU(2)$ gauge symmetry of the Lagrangian (2.1), i.e.

$$r = |\mathbf{x}| \to \infty, \quad \phi_a \to n_a\frac{m}{\sqrt{\lambda}}, \tag{2.5}$$

with n_a being an arbitrary unit vector ($|\mathbf{n}| = 1$) in the isotopic space.

One may be shown (see §10.4 in [24]) that the vacuum manifold M, i.e. the surface where the Higgs potential

$$V \equiv \frac{\lambda}{4}(\frac{m^2}{\lambda} - \phi^2)^2$$

reaches its (topologically degenerated) minimum in the isotopic space of Higgs vectors ϕ^a, is invariant under the residual gauge symmetry group $U(1) \equiv H$.

In this case the vacuum manifold M possesses the manifest geometry of the two-sphere S^2:

$$M = S^2 = \{\phi = a; \quad a^2 = m^2/\lambda\}; \tag{2.6}$$

of the radius a [4].

If we consider the maps of the sphere M, in the isotopic space of Higgs vectors, into the spatial sphere $S^2 := \{|\mathbf{n}| = 1\}$ as $\mathbf{r} \to \infty$, we obtain the chain of topological equalities:

$$\pi_2 S^2 = \pi_3(SU(2)) = \pi_1(U(1)) = \pi_1 S^1 = \mathbf{Z}. \tag{2.7}$$

Just this nontrivial topology determines magnetic charges associated with the residual $U(1)$ gauge symmetry in the Minkowskian YM theory (they themselves point to an electromagnetic theory).

In the Standard Model a non-Abelian vector field develops a mass in precisely this way. Usually, a quantum-field theory is then constructed as a perturbation theory over this vacuum in the class of function with finite energy densities.

In addition to the Minkowskian YM vacuum M, (2.6), there are monopole solutions in the system described by the Lagrangian density (2.1). These are solutions having the $O(1/r)$ type behaviour at the spatial infinity:

$$r \to \infty; \quad \phi_a - n_a \frac{m}{\sqrt{\lambda}} = O(1/r), \quad A_i^a = O(1/r). \tag{2.8}$$

A pattern of such monopole solutions is the Bogomol'nyi-Prasad-Sommerfield (BPS) solution [4], in the zero topological sector ($n = 0$), for the Minkowskian (YM-Higgs) vacuum:

$$\phi^a = \frac{x^a}{gr} f_0^{BPS}(r) \ , \qquad f_0^{BPS}(r) = \left[\frac{1}{\epsilon \tanh(r/\epsilon)} - \frac{1}{r}\right], \tag{2.9}$$

$$A_i^a(t, \mathbf{x}) \equiv \Phi_i^{aBPS}(\mathbf{x}) = \epsilon_{iak} \frac{x^k}{gr^2} f_1^{BPS}(r), \qquad f_1^{BPS} = \left[1 - \frac{r}{\epsilon \sinh(r/\epsilon)}\right], \tag{2.10}$$

which was obtained in the Bogomol'nyi-Prasad-Sommerfield (BPS) limit

$$\lambda \to 0, \qquad m \to 0; \qquad \frac{1}{\epsilon} \equiv \frac{gm}{\sqrt{\lambda}} \neq 0, \tag{2.11}$$

and is compatible with the topology (2.7) in a finite spatial volume in the Minkowski space [5].

[4]Let us denote $SU(2) \equiv G$. Since the vacuum manifold M is invariant with respect to the transformations of H, it can be subdivided onto the H-orbits of its points: $M = \sum_i HM_i$. As far as $H \subset G$, the vacuum manifold M is given as a *set of those transformations of G which do not belong to H*. Thus

$$M = G/H = \sum_i HM_i$$

and

$$G = H + G/H.$$

Such sketch is correct for each spontaneous breakdown of initial symmetry in gauge theories.

[5]The statement that the solutions (2.9), (2.10) are regular in the finite volume implies that we should consider the topology (2.7) and vacuum manifold M, (2.6), taking account of this finite spatial volume. When we wish to adapt our theory to needs of QCD (in Section 3 we shall see how to do this), the spatial volume specified by the typical hadronic size, ~ 1 fm. (~ 5 GeV^{-1}), is quite sufficient for our purposes.

The BPS vacuum solution (2.9), (2.10) is a monopole solution satisfying the YM equations and associated with the (topologically degenerated) lowest bound of the "(YM-Higgs)" energy [25] (often referred to as the *Bogomol'nyi bound*: see, e.g. [3]).

$$E_{min} = 4\pi \mathbf{m}\frac{a}{g}, \qquad\qquad a = \frac{m}{\sqrt{\lambda}}: \tag{2.12}$$

with $\mathbf{m}$ being magnetic charges associated with vacuum BPS monopoles.

In the limit specified in (2.11), the BPS solution (2.9), (2.10) involves the minimum (vacuum) "magnetic" energy that is proportional to

$$\int d^3x [B_i^a B_a^i] = 4\pi \frac{gm}{g^2\sqrt{\lambda}} = \frac{4\pi}{g^2 \epsilon}, \tag{2.13}$$

where B_i^a is the vacuum "magnetic" tension,

$$B_i^a(\Phi_k^{cBPS}) = \epsilon_{ijk}(\partial_j \Phi_k^{aBPS} + \frac{g}{2}\epsilon^{abc}\Phi_j^{bBPS}\Phi_k^{cBPS}). \tag{2.14}$$

The both formulas: (2.12), for the Bogomol'nyi bound E_{min} of the "(YM-Higgs)" energy, and (2.14), for the vacuum "magnetic" energy, are closely related each with other [25].

That the vacuum "magnetic" energy (2.14) corresponds to the Bogomol'nyi bound E_{min} of the "(YM-Higgs)" energy is ensured by the requirement that the quested vacuum "magnetic" field $\mathbf{B}$ would be of a potential character:

$$B_i^a(\Phi_k^{cBPS}) = D_i^{ab}(\Phi_k^{cBPS})\phi_b, \tag{2.15}$$

where the covariant derivative is specified by the equation (2.3). Just this condition (which ensures, as was indicated immediately above, the potential character of the vacuum "magnetic" field $\mathbf{B}$): it is referred to as the Bogomol'nyi equation [4, 25, 26], will play an important role in our construction of the stable vacuum of a non-Abelian theory (see below).

We will show now that the equation of potentiality (2.15) means a topological degeneracy of fields under the stationary gauge transformations

$$\hat{A}_i^{(n)}(t_0, \mathbf{x}) = v^{(n)}(\mathbf{x})[\hat{A}_i^{(0)}(t_0, \mathbf{x}) + \partial_i]v^{(n)}(\mathbf{x})^{-1}. \tag{2.16}$$

Dynamical fields can be represented in the form of the sum of the vacuum BPS monopole $\Phi_i^{(0)BPS}(\mathbf{x})$ and perturbations $\bar{A}_i^{(0)}$:

$$\hat{A}_i^{(0)}(t, \mathbf{x}) = \Phi_i^{(0)BPS}(\mathbf{x}) + \hat{\bar{A}}_i^{(0)}(t, \mathbf{x}). \tag{2.17}$$

Perturbations are considered as weak multipoles [26]:

$$\bar{A}_i(t, \mathbf{x})|_{assymptotics} = O(\frac{1}{r^{l+1}}) \quad (l > 1). \tag{2.18}$$

In the lowest order of the perturbation theory the equation for the temporal component of a YM field (it is *the motion equation and constraint simultaneously*),

$$[D^2(A)]^{ac}A^0_c = [D^{ac}_i(A)\partial_0 A^i_c], \tag{2.19}$$

in Dirac's variables $A_0^{Dc} = 0$ assumes the form

$$\partial_t A^{a\|}[A_i^{c(0)}(t,\mathbf{x})] = 0,$$

$$A^{a\|}[A_i^{c(0)}] \equiv [D_i^{ac}(\Phi^{(0)BPS})A_i^{c(0)}], \tag{2.20}$$

and implies that the time derivative of the longitudinal fields $A^{a\|}$ vanishes. This equation can be solved if we have initial data at our disposal. We assume that there are no longitudinal fields at the initial instant of time; that is,

$$A^{a\|} \equiv [D_i^{ac}(\Phi^{(0)BPS})A_i^{c(0)}] = 0|_{t=t_0}. \tag{2.21}$$

We refer to this condition as the covariant Coulomb gauge. There arises the question of the degree of arbitrariness in the specification of Dirac's variables associated with this gauge, since it should be recalled that they are defined apart from stationary gauge transformations.

In order to answer this question, we make these transformations:

$$\hat{A}_i^{(n)} = v^{(n)}(\hat{A}_i^{(0)} + \partial_i)v^{(n)-1}, \quad v^{(n)}(\mathbf{x}) = \exp[n\hat{\Phi}_0(\mathbf{x})], \tag{2.22}$$

and require that, upon the transformations (2.22), the fields

$$\hat{A}_i^{(n)}(t,\mathbf{x}) = \Phi_i^{(n)BPS}(\mathbf{x}) + \hat{\bar{A}}_i^{(n)}(t,\mathbf{x}) \tag{2.23}$$

satisfy the same covariant Coulomb gauge:

$$D_i^{ab}(\Phi_k^{(n)BPS})\bar{A}_b^{i(n)} = 0. \tag{2.24}$$

¿From the last condition of the gauge conservation we then obtain the so-called Gribov (ambiguity) equation [27] for the phases of gauge transformations:

$$[D_i^2(\Phi_k^{BPS})]^{ab}\Phi_{0b} = 0. \tag{2.25}$$

The Gribov equation (2.25) mathematically coincides with the Bogomol'nyi equation (2.15) for the scalar field; the latter implies the potentiality of the vacuum "magnetic" field $\mathbf{B}$ induced by vacuum BPS monopoles. Therefore, the Gribov equation (2.25) has a nontrivial solution in the form of a BPS monopole of the (2.9) type:

$$\hat{\Phi}_0 = -i\pi\frac{\tau^a x_a}{r}f_{01}^{BPS}(r), \quad f_{01}^{BPS}(r) = [\frac{1}{\tanh(r/\epsilon)} - \frac{\epsilon}{r}]. \tag{2.26}$$

Thus we have shown that vacuum YM BPS monopoles and transverse gauge physical fields belonging to the zero topological sector have their Gribov's copies in the form of the topological degeneracy (2.22).

It should be recalled that a topological degeneracy is associated primarily with a classical vacuum of zero energy, where this degeneracy is characterized by the Pontryagin index or by the Chern-Simons functional [1] (which we consider in a finite space-time of the volume V within the time interval $t_{in} < t < t_{out}$):

$$\nu[A] = \frac{g^2}{16\pi^2} \int\limits_{t_{in}}^{t_{out}} dt \int\limits_V d^3x G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a = X[A^D_{out}] - X[A^D_{in}], \tag{2.27}$$

where (see (10.93) in [24])

$$X[A] = -\frac{1}{8\pi^2} \int\limits_V d^3x \epsilon^{ijk} Tr[\hat{A}_i \partial_j \hat{A}_k - \frac{2}{3} \hat{A}_i \hat{A}_j \hat{A}_k] \tag{2.28}$$

is a topological functional of gauge fields that is reduced to an integer for a purely gauge field characterized by the degree of a map (1.6).

The functional (2.28) degenerates the quantum wave function

$$\Psi_{ins}[A] = \exp\{\pm \frac{8\pi^2}{g^2} X[A]\} \tag{2.29}$$

as an exact solution to the Schrödinger equation [2, 28]

$$\hat{H}\Psi_{ins}[A] = 0; \quad \hat{H} = \frac{1}{2} \int d^3 [\hat{E}^2 + \hat{B}^2]; \quad \hat{E} = \frac{\delta}{i\delta A}, \tag{2.30}$$

at the zero energy, $H = 0$. In just the same way as the oscillator wave function at the zero energy,

$$(\hat{p}^2 + q^2)\Psi[q] = 0,$$

this wave function is nonnormalizable, which means that the corresponding eigenenergy H belongs to unphysical values in the energy-momentum spectrum. This fact may suggest that the instanton corresponding to transitions between vacua characterized by unphysical zero values of energy is itself an unphysical solution.

Moreover, the formula (2.29) for the instanton wave function implies that the topological motion $X[A]$ is a functional of local degrees of freedom, that are denoted by A. In this case the operator of local gauge transformations

$$\hat{T} X[A] = X[A] + 1$$

does not commute with the Hamilton operator $\hat{H}$.

One of the simplest ways to remove all of these flaws, including the nonnormalizability of the wave function

$$\Psi_{ins}[A] = \exp\{iP_X X[A]\} \tag{2.31}$$

and the unphysical values of the energy and the momentum P_X of the topological motion,

$$H = 0, \quad P_X = \pm\frac{8\pi^2\ i}{g^2}, \tag{2.32}$$

consists in separating the topological motion from the local variables via the introduction of an independent topological degree of freedom $N(t)$ by means of the gauge transformation [28, 29]

$$\hat{A}_i^{(N)} = \exp[N(t)\hat{\Phi}_0(\mathbf{x})][\hat{A}_i^{(0)} + \partial_i]\exp[-N(t)\hat{\Phi}_0(\mathbf{x})]\ . \tag{2.33}$$

By means of a direct calculation, it can be proven [30] that, for the vacuum BPS monopoles $\Phi_i^{(n)}$, this degree of freedom is completely separated from the local degrees of freedom that are specified in the class of multipole functions:

$$X[A_i^{(N)}] = X[A_i^{(0)}] + N(t). \tag{2.34}$$

In this case the instanton wave function (2.31) acquires a new independent degree of freedom,

$$\Psi_{in}[A^N] = \exp\{iP_N X[A^N]\} = \exp\{iP_N(X[A^{(0)}] + N)\}, \tag{2.35}$$

and describes the topological motion of this degree of freedom at physical values of the momentum P_N. An independent topological motion arises as an inevitable consequence of a general solution to the equation $D_\mu G^{\mu 0} = 0$ for temporal YM components [28, 29]. This equation has the form

$$[D_k(A)]^{ab}[D^k(A)]_{bc}A_{0c} = D_{ci}^a(A)\partial_0 A^{ci}, \tag{2.36}$$

with the initial data being those that correspond to the vacuum BPS monopole:

$$\partial_0 A_i^c = 0; \quad A_i(t, \mathbf{x}) = \Phi_i^{BPS}(\mathbf{x}). \tag{2.37}$$

According to the theory of differential equations, the general solution to a inhomogeneous equation can be represented as the sum

$$A_0^a = \mathcal{Z}^a + \tilde{A}_0^a, \tag{2.38}$$

where $\tilde{A}_0^a$ is a particular solution to the inhomogeneous equation being considered and $\mathcal{Z}^a$ is a solution to the corresponding homogeneous equation

$$(D^2(A))^{ab}\mathcal{Z}_b = 0. \tag{2.39}$$

The phase of the topological degeneration, $\hat{\Phi}_0(\mathbf{x})$, is such solution to the homogeneous equation (2.39): apart from the factor $\dot{N}(t)$. Thus

$$\hat{\mathcal{Z}}(t, \mathbf{x}) = \dot{N}(t)\hat{\Phi}_0(\mathbf{x}). \tag{2.40}$$

The solution to the homogeneous equation (2.39) describes an "electric" monopole,

$$G_{i0}^a(\mathcal{Z}) = D_i^{ab}(\Phi^{(0)BPS})\mathcal{Z}_b = \dot{N}(t)D_i^{ab}(\Phi^{(0)BPS})\Phi_{0b}, \tag{2.41}$$

which cannot be completely removed from the action functional of the theory being considered and from the Pontryagin index by going over to Dirac's variables with the aid of gauge transformations (2.33). As was shown in [30], the Pontryagin index

$$\nu[A] = \frac{g^2}{16\pi^2} \int\limits_{t_{in}}^{t_{out}} dt \int\limits_V d^3x G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a = N(t_{out}) - N(t_{in}), \tag{2.42}$$

depends only on the difference of the final and initial values of the topological degree of freedom.

In the lowest order of the perturbation theory in the coupling constant the action functional of the theory being considered contains, in addition to vacuum YM BPS monopoles, "electric" monopoles and describes the dynamics of the new topological variable $N(t)$ in the form of a free rotator. This induces the complete action for the (YM-Higgs) Bose condensate:

$$W_{\mathcal{Z}}[N, \Phi^{BPS}] = \int dt \int\limits_V d^3x \frac{1}{2}\{[G^b_{i0}(\mathcal{Z})]^2 - [B^b_i(\Phi^{bBPS}_k)]^2\} = \int dt \frac{1}{2}\{I\dot{N}^2 - \frac{4\pi}{g^2\epsilon}\}, \tag{2.43}$$

where

$$I = \int\limits_V d^3x (D^{ac}_i(\Phi^0_k)\Phi^c_0)^2 = \frac{4\pi}{g^2}(2\pi)^2\epsilon \tag{2.44}$$

is the angular momentum of the rotator and $\epsilon = \sqrt{\lambda}/gm$ is the typical size of BPS monopoles.

The Bose condensation action obtained in such wise specifies the **Poincare invariant** Hamiltonian of the BPS monopole vacuum in terms of the canonical momentum $P_N = \dot{N}I$:

$$H = \frac{2\pi}{g^2\epsilon}[P_N^2(\frac{g^2}{8\pi^2})^2 + 1]. \tag{2.45}$$

Upon introducing new Dirac's variables with the aid of transformations (2.33), the topological degeneracy of all the fields reduces, for such Dirac's variables, to the degeneracy of only one topological variable $N(t)$ with respect to shifts of this variable on integers: $(N \Longrightarrow N + n,\ n = \pm 1, \pm 2, ...)$.

The wave function for the topological motion in the Minkowski space has the form of the free-rotator wave function

$$\Psi_{mon}[N] = \exp\{iP_N N\}, \tag{2.46}$$

with the momentum spectrum being determined from the condition $\Psi_{mon.}(N+1) = e^{i\theta}\Psi_{mon.}(N)$:

$$P_N = \dot{N}I = 2\pi k + \theta, \tag{2.47}$$

where k is the number of a Brillouin zone and θ is the angle that specifies the spectrum of physical values of the vacuum Hamiltonian (2.45). This Hamiltonian has the zero eigenvalue ($H = 0$) for the unphysical momentum values $P_N = \pm 8\pi i/g^2$, at which the instanton

wave function (2.35) coincides with the wave function (2.46) for the monopole vacuum under the assumption that the topological degree of freedom is exclusively determined by a functional of local variables.

Thus the basic distinction between the monopole vacuum and the instanton one is that in the first case there arises an independent (pseudo)Goldstone mode associated with the spontaneous breakdown of symmetry of physical states under the transformations of the $\pi_3(SU(2)) = \mathbf{Z}$ homotopies group.

The equation (2.47) and the Bogomol'nyi equation (2.15), which ensures the potential character of the vacuum "magnetic" field $\mathbf{B}$, determine the spectrum of the vacuum "electric" field ("electric" monopole):

$$G^a_{i0} = \dot{N}(t)\ (D_i(\Phi^0_k)\Phi_0)^a = P_N \frac{\alpha_s}{\pi^2\epsilon} B^a_i(\Phi_0) = |2\pi k + \theta| \frac{\alpha_s}{\pi^2\epsilon} B^a_i(\Phi_0). \tag{2.48}$$

The expression (2.48) is an analogue of the spectrum of the electric field tension

$$G_{10} = e(\frac{\theta}{2\pi} + k)$$

in two-dimensional electrodynamics [31, 32, 33] characterized by the same topology of degeneracy of initial data:

$$\pi_1(U(1)) = \pi_3(SU(2)) = \mathbf{Z}.$$

The vanishing of the topological momentum does not imply that the degeneracy of physical states disappears. Physical implications of such a degeneracy will be considered in the next section.

3 Yang-Mils theory featuring topological degeneracy of physical states.

It is well known that the perturbation theory constructed for non-Abelian fields by analogy with QCD [10, 17] is infrared-unstable [34, 35]. A conventional vacuum of the perturbation theory, $A = 0$, is not a stable state. As a rule, homogeneous ([34]) or singular fields, including instantons ([1, 25]), the equation for which involves delta-function-like singularities in the Euclidean space E_4, are used for describing nonzero vacuum fields. If one explains physical effects by a homogeneous (or by an instanton) vacuum, it is also necessary to explain the emergence of capacitors at the spatial infinity that generate homogeneous fields (or the origin of sources of delta-function-like singularities) [6].

Unlike, BPS monopoles provide a unique possibility for introducing, in the class of regular functions associated with topologically nontrivial gauge transformations, vacuum

[6] We recommend our readers the interpretation of sources with the help of delta-function-like singularities given in the monograph [36]: in §11.3.

fields in such a way that the equations of the Yang -Mills theory (it is (1.1) in our case) do not develop any additional sources.

In order to introduce such a monopole vacuum, we include the interaction of gauge fields with a Higgs field in a space of the finite volume $V = \int d^3x$. The scalar-field condensate forms a BPS monopole characterized by a finite mass of the scalar field:

$$\frac{1}{\epsilon} = \frac{gm}{\sqrt{\lambda}} = \frac{g^2 < B^2 > V}{4\pi}. \tag{3.1}$$

If we go over to the infinite-volume limit $V \to \infty$ under the condition that $< B^2 >$ is finite, the scalar field acquires an infinitely large mass and disappears from the spectrum of physical excitations, while the regular solution represented by a vacuum BPS monopole continuously transforms into a Wu-Yang monopole [37]; this means that the equations of the theory do not develop, at the origin of coordinates, a singularity inherent in Wu-Yang monopoles [37] and that the energy density does not go to infinity. In this limit the finite energy density of the appropriate BPS monopole has the form

$$\int d^3x [B_i^a]^2 \equiv V < B^2 >, \tag{3.2}$$

where $< B^2 >$ is the quantity that one has for the order parameter of the physical vacuum of the gauge field upon the removal of the scalar field.

We recall that a Wu-Yang monopole [37] is an exact solution to the classical equations of the "pure" YM theory, (1.3), everywhere, with the exception of an infinite-small neighbourhood of the origin of coordinates. Just in this small region around the origin of coordinates Wu-Yang monopoles are regularized by the scalar-field mass and this region disappears in the infinite-volume limit $V \to \infty$ in (3.2).

It should be noted here that, in quantum field theories, a transition to the limit $V \to \infty$ is performed upon calculating physical observables, such as scattering cross sections and decay probabilities, that are normalized per unit time and per unit volume. Therefore, all the specific features of the above theory involving vacuum BPS monopoles, which include the topological degeneracy of initial data and "electric" monopoles, survive at any finite value of the volume.

On the other hand, there are, in the YM theory specified by the Lagrangian density (1.1), direct indications that the scale (gauge) symmetry of the vacuum is broken by solutions belonging to the type of a Wu-Yang monopole [37]. In particular, the topological classification of the classical solutions to the "pure" YM theory specifies the class of solutions that possess the zero topological index ($n = 0$),

$$X[A = \Phi^{(0)}] = 0, \quad \frac{\delta X[A]}{\delta A_i^c}|_{A=\Phi^{(0)}} \neq 0, \tag{3.3}$$

and which have the form

$$\hat{\Phi}_i^{(0)} = -i\frac{\tau^a}{2}\epsilon_{iak}\frac{x^k}{r^2}f(r), \tag{3.4}$$

where there is only the one unknown function, $f(r)$. The equation for this function can be obtained by substituting the expression (3.4) into the classical equation (1.3):

$$D_k^{ab}(\Phi_i^{(0)})G_b^{kj}(\Phi_i^{(0)}) = 0 \Longrightarrow \frac{d^2 f}{dr^2} + \frac{f(f^2-1)}{r^2} = 0. \tag{3.5}$$

In the region $r \neq 0$ there exist the following three solutions to this equation:

$$f_1^{PT} = 0, \quad f_1^{WY} = \pm 1. \tag{3.6}$$

The first, trivial, solution $f_1^{PT} = 0$ corresponds to the ordinary unstable perturbation theory involving the "asymptotic freedom " [34, 35]. Two nontrivial solutions $f_1^{WY} = \pm 1$ represent Wu-Yang monopoles, which, in the model being considered, emerge from (vacuum) BPS monopoles in the infinite-volume limit without their singularities, with (pseudo)Goldstone modes accompanying the breakdown of the scale invariance.

Thus the monopole vacuum characterized by a topological degeneracy of all the physical states has the following features distinguishing it from the topologically degenerate instanton vacuum [1, 3, 25]: the Minkowski space; topological (pseudo)Goldstone modes associated with the scale-symmetry breaking that generates the nonzero order parameter $< B^2 > \neq 0$ and a clear physical origin of the scale-symmetry breaking, which is due to the condensate of the scalar Higgs field and which survives upon removing the scalar field from the excitation spectrum.

Physical implications of the theory being considered, which involves the YM Minkowskian monopole vacuum, are controlled by the generating functional for the unitary perturbation theory in the covariant Coulomb gauge. Reproducing the calculations performed in the work [17] (also see [18]), one can obtain, as the generating functional for such a perturbation theory, a Feynman path integral in a reference frame with a specified time axis, $l_\mu = (1, 0, 0, 0)$:

$$Z^*[l, J^*] = \int\int \prod_t dN(t) \prod_{c=1}^{c=3} [d^2 A^{*c} d^2 E^{*c}] \exp\{iW^*[N, A^*, E^*] + i\int d^4 x J^* \cdot A^*\}, \tag{3.7}$$

where A^* are Dirac's variables (2.33); E^* are their canonically conjugate momenta; J^* are their sources; $W^*[N, A^*, E^*]$ is the original action functional taken on the manifold spanned by solutions to the constraint equation,

$$\frac{\delta W}{\delta A_0} = 0, \Longrightarrow D_i^{cd}(A) G_{d0}^i = 0, \tag{3.8}$$

for the non-Abelian electric-field tension G_{0i}^d represented in the form of the sum of the transverse momentum E^* and the longitudinal component:

$$G_{0i}^d = E_i^{*d} + D_i^{db}(\Phi)\sigma_b \quad (D_i^{cd}(\Phi^{N(WY)}) E_d^{*i} = 0). \tag{3.9}$$

If one assumes that, in the perturbation theory, independent Dirac's variables (2.33), $A_i^{*d} = \Phi_i^{dN(WY)} + \bar{A}_i^{*d}$, satisfy the gauge condition

$$D_i^{cd}(\Phi^{N(WY)} A_d^{*i} = 0, \tag{3.10}$$

which were considered above, then the constraint equation (3.8) reduces to an equation for the function σ^b:

$$D_i^{cd}(A^*)D_{db}^i(\Phi^{N(WY)})\sigma^b = j_0^c, \tag{3.11}$$

where the quantity in the right-hand side is the current of independent non-Abelian variables,

$$j_0^a = g\epsilon^{abc}[A_i^{*b} - \Phi_i^{b\ N(WY)}]\tilde{E}_c^{*i}, \tag{3.12}$$

belonging to the excitations spectrum.

One can solve the equation (3.11), which involves transverse quantum excitations $\tilde{E}_*$ over the zero mode (described above), by means of the perturbation theory, employing a Green's function of the Coulomb type. In the lowest order of the perturbation theory this Green's function $G^{bc}(\mathbf{x},\mathbf{y})$ in the field of an usual Wu-Yang monopole Φ^{WY} is determined by the equation

$$[D^2(\Phi^{WY})]^{ab}(\mathbf{x})G_b^c(\mathbf{x},\mathbf{y}) = \delta^{ac}\delta^3(\mathbf{x}-\mathbf{y}). \tag{3.13}$$

A solution to this equation specifies, in the Hamiltonian, an instantaneous interaction of non-Abelian currents,

$$-\frac{1}{2}\int_{V_0} d^3x d^3y j_0^b(\mathbf{x})G_{bc}(\mathbf{x},\mathbf{y})j_0^c(\mathbf{y}), \tag{3.14}$$

as an analogue of the Coulomb interaction of the currents in QED. A solution to the equation (3.13) in the presence a Wu-Yang monopole, where

$$[D^2(\Phi^{WY})]^{ab}(\mathbf{x}) = \delta^{ab}\Delta - \frac{n^a n^b + \delta^{ab}}{r^2} + 2(\frac{n^a}{r}\partial^b - \frac{n^b}{r}\partial^a); \tag{3.15}$$

$n_a(x) = x_a/r$; $r = |\mathbf{x}|$, was obtained in the works [12, 38] by means of an expansion of G^{ab} in terms of a complete set of orthogonal vectors:

$$G^{ab}(\mathbf{x},\mathbf{y}) = [n^a(\mathbf{x})n^b(\mathbf{y})V_0(z) + \sum_{\alpha=1,2} e_\alpha^a(x)e^{b\alpha}(y)V_1(z)]; \quad z = |\mathbf{x}-\mathbf{y}|, \tag{3.16}$$

where $V_{0,1}(z)$ are potentials. Substituting this expansion into the equation (3.13), one can derive an equation for the potentials. The result is

$$\frac{d^2}{dz}V_n + \frac{2}{z}\frac{d}{dz}V_n - \frac{n}{z^2}V_n = 0; \quad n = 0,1. \tag{3.17}$$

Solving this equation, we obtain the potentials

$$V_n(|\mathbf{x}-\mathbf{y}|) = d_n|\mathbf{x}-\mathbf{y}|^{l_1^n} + c_n|\mathbf{x}-\mathbf{y}|^{l_2^n}; \quad n = 0,1, \tag{3.18}$$

where d_n and c_n are constants, while l_1^n and l_2^n are the roots of the equation $(l^n)^2 + l^n = n$, i.e.

$$l_1^n = -\frac{1+\sqrt{1+4n}}{2}; \qquad l_2^n = \frac{-1+\sqrt{1+4n}}{2}. \tag{3.19}$$

At $n = 0$ we have $l_1^0 = -(1+\sqrt{1})/2 = -1$ and $l_2^0 = (-1+\sqrt{1})/2 = 0$, so that there arises the Coulomb potential

$$V_0(|\mathbf{x}-\mathbf{y}|) = -1/4\pi|\mathbf{x}-\mathbf{y}|^{-1} + c_0; \tag{3.20}$$

at $n = 1$, $l_1^1 = -(1+\sqrt{5})/2 \approx -1.618$ and $l_2^1 = (-1+\sqrt{5})/2 \approx 0.618$, in which case one get the rising potential for the golden-section equation $(l^1)^2 + l^1 = 1$:

$$V_1(|\mathbf{x}-\mathbf{y}|) = -d_1|\mathbf{x}-\mathbf{y}|^{-1.618} + c_1|\mathbf{x}-\mathbf{y}|^{0.618}. \tag{3.21}$$

As was shown in the works [39, 40, 41], the instantaneous interaction of colour currents through a rising potential rearranges perturbation-theory series and leads to the constituent mass of the gluon field in Feynman diagrams; this changes the asymptotic-freedom formula at low momentum transferred, so that the coupling constant $\alpha_{QCD}(q^2 \sim 0)$ becomes finite. The rising potentials of the instantaneous interactions of colour currents [39, 40, 41] also lead to a spontaneous breakdown of the chiral invariance for quarks.

Rising potentials do not remove poles of Green's functions in a perturbation theory for amplitudes of processes not involving colour degrees of freedom. Such a perturbation theory is formulated in terms of fields that are characterized by zero topological quantum numbers and that can be called partons:

$$\hat{A}^*(N|A^{(0)}) = U_N[\hat{A}^{(0)} + \partial]U_N^{-1}. \tag{3.22}$$

By virtue of the gauge invariance, the phase factors of the topological degeneracy,

$$U_N = \exp\{N(t)\hat{\Phi}_0(\mathbf{x})\},$$

disappear. However, these factors survive at the sources of physical fields in the generating functional (3.7). A theory featuring a topological degeneracy of initial data, where the sources of physical fields involve the Gribov factors

$$\mathrm{tr}[\hat{J}^i v^{(n)} \bar{\hat{A}}_i^{(0)} v^{(n)-1}]$$

differs from a theory that is free from any degeneracy and that involves the sources $\mathrm{tr}[\hat{J}^i \bar{\hat{A}}_i^{(0)}]$. In a theory featuring a degeneracy of initial data it is necessary to average amplitudes over degeneracy parameters. Such averaging may lead to the disappearance of a number of physical states.

In [29, 42] it was shown that amplitudes for the production of physical colour particles may vanish because of the destructive interference between the phase factors of the topological degeneracy. In this case the probability-conservation law for the S-matrix elements $< i|S = I + iT|j >$ in the form

$$\sum_f < i|T|f >< f|T^*|j >= 2\ \mathrm{Im} < i|T|j >$$

is exclusively saturated by the production of colour-singlet states (hadrons) $f = h$. By virtue of the probability-conservation law, the sum over all the hadronic channels becomes equal to the doubled imaginary part of the colour-singlet amplitude 2 Im $< i|T|j >$.

In turn, the dependence on the factors of the topological degeneracy completely disappears in the colour-singlet amplitude. Owing to the gauge invariance, the Hamiltonian of the theory, $H[A^{(n)}] = H[A^{(0)}]$, depends only on the fields of the zero topological sector, $A^{(0)}$, which play the role of Feynman 's partons. In the high-energy parton region, where the imaginary part of the colour-singlet amplitude, Im $< i|T|j >$, can be calculated on the basis of the perturbation theory, the quark-hadron duality, which is used to directly measure parton quantum numbers coinciding with the quantum numbers of physical colour particles, arises from the probability-conservation law.

4 Estimating quantity $< B^2 >$ within QCD.

Let us estimate the quantity $< B^2 >$ within QCD.

In the monopole vacuum of QCD the antisymmetric Gell-Mann matrices $\lambda_2, \lambda_5, \lambda_7$ play the role of the matrices τ_1, τ_2, τ_3.

The instantaneous interaction of colour quark currents through a rising potential leads to the spectrum of mesons: in particular, to the pseudoscalar η_0 meson. Its anomalous interaction with gluons is described in terms of the Veneziano effective action [43]

$$W_{eff} = \int dt \left\{ \frac{1}{2} \left(\dot{\eta}_0^2 - M_0^2 {\eta_0}^2 \right) V + C_0 \eta_0 \dot{X}[A^{(N)}] \right\}, \tag{4.1}$$

in the rest frame of this meson. Here V is the spatial volume; $C_0 = (N_f/F_\pi)\sqrt{2}/\pi$ is the coupling constant for the anomalous interaction of the meson with the topological functional $X[A^{(N)}] = X[A] + N$; F_π is the weak-pion-decay constant and $N_f = 3$ is the number of flavours.

The calculation of a similar action functional for QCD and for QED$_{(3+1)}$, where the topological functional describes the decay of a parapositronium into two photons, is represented in the work [12].

In all probability, the expression (4.1) for the effective anomalous interaction of a pseudoscalar state with gauge fields is common for all the gauge theories.

For electrodynamics in the two-dimensional space-time one can obtain the same effective action [32, 33], where it leads to the mass of the Schwinger bound state.

In QCD$_{(3+1)}$ the extra mass of a bound pseudoscalar state,

$$\Delta m_\eta^2 = C_0^2 / I_{QCD} V,$$

can be determined, upon adding the action functional that is specified by the formulas (2.43), (2.44) and that controls the topological dynamics of the zero mode,

$$W_{QCD} = \frac{1}{2} \int dt \int\limits_V d^3x G_{0i}^2 = \int dt \frac{\dot{N}^2 I_{QCD}}{2} \quad \left(I_{QCD} = \left(\frac{2\pi}{\alpha_{QCD}} \right)^2 \frac{1}{V < B^2 >} \right), \tag{4.2}$$

to the anomalous Veneziano action, by diagonalizing the total Lagrangian:

$$L = [\frac{\dot{N}^2 I_{QCD}}{2} + C_0\eta_0\dot{N}] = [\frac{(\dot{N} + C_0\eta_0/I_{QCD})^2 I_{QCD}}{2} - \frac{C_0^2}{2I_{QCD}}\eta_0^2]. \tag{4.3}$$

In QED$_{(1+1)}$ the analogous formula describes the mass of the Schwinger state [32, 33], whereas in QCD$_{(3+1)}$ we obtain the extra mass of the η_0 meson:

$$L_{\text{eff}} = \frac{1}{2}[\dot{\eta}_0^2 - \eta_0^2(t)(m_0^2 + \triangle m_\eta^2)]V, \tag{4.4}$$

$$\triangle m_\eta{}^2 = \frac{C_\eta^2}{I_{QCD}V} = \frac{N_f^2}{F_\pi^2}\frac{\alpha_{QCD}^2 < B^2 >}{2\pi^3}. \tag{4.5}$$

This result makes it possible to assess the vacuum expectation value of the chromomagnetic field in QCD$_{(3+1)}$:

$$< B^2 > \alpha_{QCD}^2 = \frac{2\pi^3 F_\pi^2 \triangle m_\eta^2}{N_f^2} = 0.06 \text{ GeV}^4,$$

by using the estimate $\alpha_{QCD}(q^2 \sim 0) \sim 0.24$ [39, 44].

Upon the calculation, we can remove infrared regularization by going over to the limit $V \to \infty$.

Conclusion.

The monopole-vacuum model considered here demonstrates that the quantization of a non-Abelian theory featuring a topological degeneracy of the initial data for all the physical states in a specific reference frame describes a destructive interference of degeneracy phase factors, which leads to the quark-hadron duality; a (pseudo)Goldstone mode that is associated with a spontaneous breakdown of the initial gauge symmetry and which leads to an extra mass of the η_0 meson; a rising potential that controls the instantaneous interaction of currents and which is thought to be responsible for a spontaneous breakdown of the chiral symmetry. These hidden features of non-Abelian fields manifest themselves upon switching on and off the gauge field interaction with a Higgs field, which acquires an infinitely large mass in the infinite-volume limit. There arises the question of the extent to which such a fantastic possibility may be realized in nature.

The generating functional found in the form of a Feynman path integral with respect to Dirac's variables can be recast [17] into the form of a FP integral [23] by means of the change of variables

$$\hat{A}_i^*(N|A) = (U_n U^D[A])[\hat{A}_i + \partial_i](U_n U^D[A])^{-1},$$

where [12]:

$$U^D[A] = \exp\{\frac{1}{D^2(\Phi^{WY})}D_k(\Phi^{WY})\hat{A}^k\} \tag{C.1}$$

is Dirac's "dressing" of non-Abelian fields. Upon this change of variables, the Feynman path integral is reduced to the FP integral

$$
\begin{aligned}
Z[l, J^*] \;\; &= \;\; \int \prod_t dN(t) \int \int \prod_{c=1}^{c=3} [d^4 A^c] \delta(f(A)) \; \text{Det} \; M_{FP} \\
&\times \exp \left\{ iW[A] + i \int d^4 x J^* \cdot A^*(N|A) \right\}
\end{aligned}
\tag{C.2}
$$

in an arbitrary gauge of the physical variables, $f(A) = 0$; here the FP determinant Det M_{FP} is determined in terms of the linear response of this gauge to a gauge transformation, $f(e^{\Omega}(A + \partial)e^{-\Omega} = M_{FP}\Omega + O(\Omega^2)$, while W is the original action functional in the theory being considered. At the same time, there remains the Dirac gauge of the sources in (C.1).

As a relic of the fundamental quantization, the Dirac phase factors in the integral in (C.2) "remember " the entire body of information about the reference frame; monopoles; the rising potential of the instantaneous interaction and other initial data, including their topological degeneracy and confinement.

As was predicted by Schwinger [10], all these effects disappear, leaving no trace, if these Dirac factors are removed by means of the replacement $A^*(N|A) \Longrightarrow A$ [17], which is made with the only purpose of removing the dependence of the path integral on a reference frame and initial data. On getting rid of this dependence, we obtain, instead of hadronization and confinement in the Dirac's quantization scheme, only the amplitudes for the scattering of free partons in the "relativistic " FP integral, which do not exist as physical observables in the initial-data-dependent Dirac's scheme of quantization.

The same metamorphosis occurs in QED as well: going over from the Dirac gauge of sources to the Lorentz gauge in order to remove the dependence on a reference frame and initial data, we replace the perturbation theory emerging upon the fundamental quantization and featuring two singularities in the photon propagators (a single-time singularity and that at the light cone) by the perturbation theory in the Lorentz gauge; the latter involves only one singularity in the propagators (that at the light cone), but, in principle, cannot describe single-time Coulomb atoms, containing only Wick-Cutkosky bound states whose spectrum in not observed in nature [45].

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